Multi-objective Linear Programming Optimization for Waste Management Simulation

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Abstract—**Municipal Solid Waste Management (MSWM) is a very complex problem present in many communities around the world. Decision makers need to formulate solutions that consider multiple goals and strategies. Strategies include multiple options for waste collection, transportation, transfer, treatment and disposal. The most appropriate choice, however, is often not clear. Given the large number of available options for MSWM and the interrelationships among these options, identifying MSWM strategies that satisfy economic or environmental objectives is a complex task. The main objective of this work is to use MSWM simulation and multi-objective linear programming to support decision makers in the process of selecting MSWM strategies from a very large decision space and in the presence of multiple objectives. Three competing objectives are considered: least cost, minimization of carbon dioxide emissions and minimization of energy consumption. A multi-objective fuzzy linear programming method is proposed to manage imprecision and uncertainty in the objectives via fuzzy membership functions.**

Keywords-simulation model; linear programming; fuzzy multiobjective optimization; Pareto optimal set; waste management.

I. INTRODUCTION

Municipal Solid Waste Management (MSWM) is getting increased attention at national and local levels. The specific goals of each community for implementing MSWM plans depend on site-specific conditions and issues. For instance, a community facing a landfill space crisis may set a goal to reduce the amount of waste sent to landfill disposal and may consider source reduction, waste diversion through recycling, and volume reduction alternatives such as converting waste to energy. The most appropriate choice, however, is often not clear. For instance, recycling is known to reduce consumption of natural resources and save some processing activities at manufacturing facilities. These savings will avoid the emissions of some associated greenhouse gases and pollutants. However, if the market prices of recyclable materials are low, then a recycling program may not be as economical as one of the other options. To add complexity to this problem, landfill space may be very limited, making recycling an attractive option regardless of low market prices for recyclable materials. Each step in waste management (collection, recycling, treatment, disposal) could be accomplished through different technological options, and an overall MSWM overall strategy should include in most cases at least one technological option for each step. Decision makers are then faced with a problem of multiple dimensions: they need to select from multiple technological options to manage municipal waste from the generation point to the final disposal point to create an overall strategy, and they must evaluate each overall strategy for the competing objectives of cost effectiveness and environmental impact reduction. Given the large number of available options for MSWM and the interrelationships among these options, identifying MSWM strategies that satisfy economic or environmental objectives is a complex task. Simulation of MSWM is used in this work to help the decision makers with the screening and identification of MSWM strategies.

The main objective of this work is to use MSWM simulation and multi-objective linear programming to select MSWM strategies from a very large decision space. Multiple objectives are considered: least cost, minimization of carbon dioxide emissions and minimization of energy consumption. Least cost solutions will tend to select MSWM technological options that do not necessarily reduce emissions or energy consumption. Conversely, minimum energy consumption scenarios and minimum pollutant emission scenarios will tend to select more advanced and expensive MSWM technological options. A number of methodologies have been proposed for multi-objective linear programming, such as the ones reported in [4], [5], [6], [7], [8], [9], and [10]. None of these methodologies offers a simple way to evaluate the relative importance of objective functions with very dissimilar measurement units. A new methodology using MSWM simulation and multi-objective fuzzy linear programming is proposed to improve the traditional weighted sum methodology. It is used to select noninferior solutions and to quantify the degree of achievement for each of the competing objectives.

II. METHODOLOGY

A. *Municipal Solid Waste Management Simulation*

Simulation of MSWM can be performed by using a systemic representation of the different MSWM options. Each MSWM option or node has a specific purpose (e.g.

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waste collection, waste transfer or waste treatment) and is inter-related with a number of other MSWM options. Waste processes are simulated as mass balances at each management option, where there is incoming waste from other nodes and outgoing waste to other nodes. This simulation includes a number of sub-models, one for each MSWM option. These sub-models use the Life Cycle Inventory methodology to calculate annualized cost, energy consumption and emissions of different pollutants. The mathematical formulation is linear programming (LP) in which the objective functions are to minimize cost, energy consumption or a number of pollutant emissions. The simulation includes a life cycle inventory of different pollutants and the user can select to minimize the emissions of any of these pollutants. It also includes a complex mathematical formulation via constraints to represent waste mass flows from process units to other process units. For more details on the model simulation design and mathematical formulation see references [1], [2] and [3]. Table 1 shows a list of MSWM options used in this study.

TABLE I. MUNICIPAL SOLID WASTE MANAGEMENT OPTIONS

MSWM step	*Management Option*
Collection	Residential Collection of Yardwaste
Collection	Residential Collection of Mixed MSW
Collection	Residential Collection of Commingled Recyclables Sorted by Crew
Collection	Residential Collection of Presorted Recyclables
Collection	Residential Collection of Commingled Recyclables Sorted at Materials Recovery Facility
MRF	Materials Recovery Facility (MRF) for Mixed MSW
MRF	Materials Recovery Facility (MRF) for Presorted Recyclables
MRF	Materials Recovery Facility (MRF) for Commingled Recyclables
Treatment	Treatment at Yardwaste Compost Plant
Treatment	Treatment at Mixed MSW Combustion Plant
Treatment	Treatment at Refuse Derived Fuel (RDF) Plant
Disposal	Disposal at Landfill
Disposal	Disposal at Ash-landfill

B. *Generation of Baseline Solutions*

A hypothetical case representing a residential urban region of medium size will be defined. The residential population is 400,000 people with a solid waste generation rate of 4.5 lbs/person-day. The MSWM system definition requires specification of many input parameters, e.g., waste composition, distances between waste processing facilities, collection frequencies, etc. Most of these input parameters use national average values. The model was applied to an illustrative problem scenario where 15% of overall generated waste was banned from being disposed at the landfill. Diverting waste to be disposed at the local landfill can be achieved via yardwaste composting and recycling programs. The diversion rate will be enforced by adding a diversion constraint in addition to the mass flow constraints. Single objective simulations were executed to obtain the minimum-cost solution, the minimum carbon dioxide emissions solution and the minimum energy consumption solution. The resulting MSWM options selected from the decision space were reported for each case. The resulting cost or environmental objective values selected from the objective space were reported for each case.

C. *Generation of Pareto Optimal Set*

The next step was to generate additional MSWM solutions using multi-objective programming. The three solutions obtained previously were generated by minimizing on one of the main objectives at a time. Since these objectives can be conflicting with one other, we are now interested in formulating a single aggregate objective function that incorporates all three objectives. This is done by creating a weighted linear sum of the objectives.

Equation 1 shows the weighted linear sum of the three objectives: cost, energy consumption and carbon dioxide emissions. This objective function is optimized subject to the mass flows constraints and the diversion constraint. The objective function weights w_{cost}, w_{CO2} and w_{enegy} are supplied to obtain different optional solutions. They dictate how much of one objective must be sacrificed for the benefit of the other objectives. All solutions obtained with this method constitute non-inferior solutions necessary to generate the Pareto Optimal Set.

$$\text{Min}\ \{w_{cost} \text{ x } Z_1 + w_{energy} \text{ x } Z_2 + w_{CO2} \text{ x } Z_3\}. \tag{1}$$

Subject to: mass flow and waste diversion constraints.

where: Z_1 is Cost, Z_2 is Energy Consumption and Z_3 is CO_2 emissions. Weights w_{cost}, w_{CO2} and w_{enegy} are greater than zero and their sum is equal to 1.

D. *Fuzzy Linear Programming*

The last step was to use fuzzy linear programming to represent the objective functions as fuzzy sets. The use of fuzzy sets tries to capture the imprecision and uncertainty of competing objectives. A number of methodologies have been proposed for fuzzy multi-objective linear programming, such as the ones reported in [4], [5], [6], [7], [8], [9] and [10]. The method proposed here is a modification of the method proposed by Raju and Kumar [4]. The relative importance of the objectives is measured by the membership functions. The membership function is a measure of the degree of achievement for any given objective and is represented by $\mu_i(X)$ in Equation 2.

$$\mu_i(X) = \begin{cases} 0, \text{ for } Z_i < Z_{L,i} \\ (Z_i - Z_{L,i})/(Z_{U,i} - Z_{L,i}), \text{ for } Z_i < Z_{L,i} \\ 1, \text{ for } Z_i > Z_{U,i} \end{cases} \tag{2}$$

where:
$Z_{L,i}$: less desirable value for objective i
$Z_{U,i}$: most desirable value for objective i
Z_i: objective value linked to degree of achievement μ_i
X: decision variables vector

Equation 1 can be re-written as:

$$\text{Min } \lambda \tag{3}$$

Subject to:

$$\lambda = w_{cost} \times \lambda_1 + w_{energy} \times \lambda_2 + w_{CO2} \times \lambda_3$$
$$(Z_{cost} - Z_{L,cost})/(Z_{U,cost} - Z_{L,cost}) \leq \lambda_1$$
$$(Z_{energy} - Z_{L,energy})/(Z_{U,energy} - Z_{L,energy}) \leq \lambda_2$$
$$(Z_{CO2} - Z_{L,CO2})/(Z_{U,CO2} - Z_{L,CO2}) \leq \lambda_3$$
$$0 \leq \lambda_1 \leq 1,\ 0 \leq \lambda_2 \leq 1,\ 0 \leq \lambda_3 \leq 1$$

mass flow and waste diversion constraints

A value of 0 for λ_1 means that there was a perfect achievement to minimize the cost objective, and a value of 1 means that the worst achievement for cost was obtained. The same applies for λ_2 and the energy objective and for λ_3 and the CO_2 objective. The degree of achievement can be then defined as 1- λ_i. Under this modified formulation, weights w_{cost}, w_{CO2} and w_{enegy} have values greater than zero but do not need to add up to 1. They measure the objectives relative importance between one another.

III. RESULTS AND ANALYSIS

A. *Minimum Cost Solution*

The model simulation resulted in a minimum cost solution that includes yard waste collection and composting for a sector of the residential population (23,800 tons). It also includes collection of mixed MSW taken to a mixed MSW MRF for separation of recyclable materials (260,000 tons).

Recyclable materials sorted and processed (25,500 tons) at the mixed waste MRF include among others: old newspaper (10,500 tons), corrugated cardboard (3,280 tons), ferrous cans (2,350), plastic (782 tons), clear glass (6,100) and brown glass (2,500 tons). This amount of composted and recycled material helped comply with the mandatory 15% diversion policy goal. Fig. 1 shows the selection of technologies for the minimum cost solution. The minimum cost obtained was $33 million, with -40 million pounds of CO_2 emissions and -0.18 trillion BTU of energy consumption. A negative value in CO_2 emissions indicates that CO_2 emissions generated by unit processes (35 million lbs) are offset by the remanufacturing of recyclable materials into other usable materials (avoided 32 million lbs). These remanufacturing processes use recyclable material instead of virgin materials. Avoided emissions from the extraction and processing of raw materials are accounted for in the life cycle inventory. Additional emission offsets are accounted for at the landfill where CO_2 is sequestered and avoided from entering the atmosphere (43 million lbs). Similarly, a negative value in the energy consumption means that energy consumed by the unit processes (0.27 trillion BTU) is offset by the remanufacturing process (avoided 0.29 trillion BTU). Avoided energy consumption from the extraction and processing of raw materials is accounted for in the life cycle inventory.

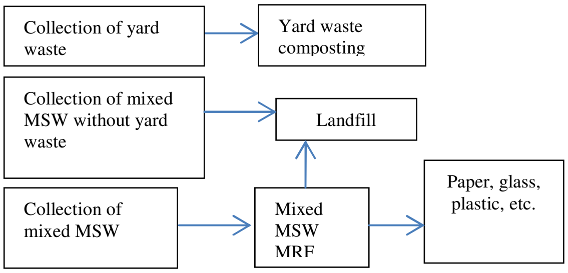

Figure 1. Management options for minimum cost.

B. *Minimum Carbon Dioxide Emissions Solution*

The minimum carbon dioxide emissions solution includes quite different options from the minimum cost solution. It includes collection of recyclables sorted by the collection crew and taken to a presorted recyclables MRF (29,200 tons). It also includes collection of mixed MSW taken to a mixed MSW MRF (299,000 tons). Residuals from the mixed MSW MRF are then taken to a waste to energy facility (280,000 tons). Recyclable materials sorted and processed at the mixed MSW MRF (18,800 tons) include among others: old newspaper (6,830 tons) and clear glass (4,160 tons); and recyclable materials sorted at the presorted recyclables MRF (29,200 tons) include among others: old newspaper (9,590 tons) and clear glass (5,250 tons). Fig. 2 shows the selection of technologies for the minimum carbon dioxide emissions solution. The minimum carbon dioxide obtained was -202 million pounds of CO_2 (offset), with a cost of $57.6 million and -2.2 trillion BTU of energy consumption (offset).

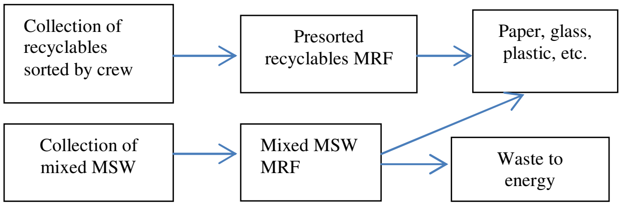

Figure 2. Management options for minimum carbon dioxide emissions.

C. *Minimum Energy Consumption Solution*

The minimum energy consumption solution includes collection of commingled recyclables taken to a commingled recyclables MRF (27,600 tons). It also includes collection of mixed MSW taken to a mixed MSW MRF (301,000 tons). Residuals from the mixed MSW MRF are then taken to a waste to energy facility (280,000 tons). Recyclable materials sorted and processed at the commingled materials MRF (27,600 tons) include among others: old newspaper (9,590 tons) and clear glass (5,250 tons); and recyclable materials sorted at the mixed waste MRF (21,000 tons) include among others: old newspaper (6,830 tons) and clear glass (4,160 tons). Fig. 3 shows the selection of technologies for the minimum energy consumption solution. The minimum energy consumption obtained was -2.27 trillion BTU (offset), with a cost of $60.3 million and -198 million pounds of CO_2 emissions (offset).

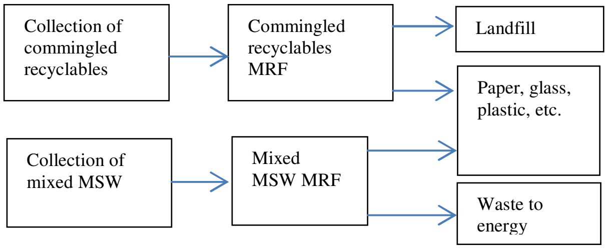

Figure 3. Management Options for minimum energy consumption.

D. *Multi-objective Optimization*

Additional solutions were found by using multi-objective optimization and the weighted sum method. Table 2 shows the weight values used for each multi-objective formulation, and summarizes the values of the three objectives for each of the noninferior solutions. Weights were varied arbitrarily to try to capture different solutions. The optimal solution obtained depended on the relative values of the used weights. For example, if the specified weight for the cost objective was greater than the specified weight for the energy consumption objective, the solution favored lower cost over lower energy consumption.

Solution A corresponds to the least cost solution (Z1*); solution E to the minimum CO_2 emissions solution (Z3*); and solution F to the minimum energy solution (Z2*). The other solutions represent noninferior points in the multi-objective solution space.

TABLE II. SUMMARY OF SOLUTIONS FROM MULTI-OBJECTIVE OPTIMIZATION

Sol.	w_{cost}	w_{energy}	w_{CO2}	***Cost (10^6 $)***	***Energy (trillion BTU)***	***CO_2 (10^6 lbs)***
A	0.999	5e-4	5e-4	33.1 (Z1*)	0.018	39.8
B	0.8	0.1	0.1	36.4	1.207	137.0
C	0.6	0.2	0.2	45.6	2.070	187.0
D	0.4	0.3	0.3	52.1	2.203	196.0
E	0.2	0.4	0.4	57.6	2.205	202.4 (Z3*)
F	5e-5	0.9999	5e-5	59.9	2.270 (Z2*)	195.9

Fig. 4 shows a plot of the multi-objective solutions as a tradeoff between the conflicting objectives of least cost and minimum CO_2 emissions. Similarly, Fig. 5 shows a plot of the multi-objective solutions as a tradeoff between the conflicting objectives of least cost and minimum energy consumption. These tradeoff curves represent the Pareto optimal sets or frontiers for the conflicting objectives. Any point in the Pareto optimal set represents a noninferior solution, for which an improvement in one objective requires a degradation of the other. Energy and emissions are non-conflicting objectives.

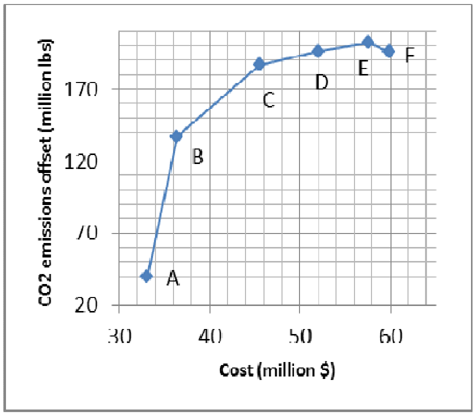

Figure 4. Cost and CO_2 emissions tradeoff curve.

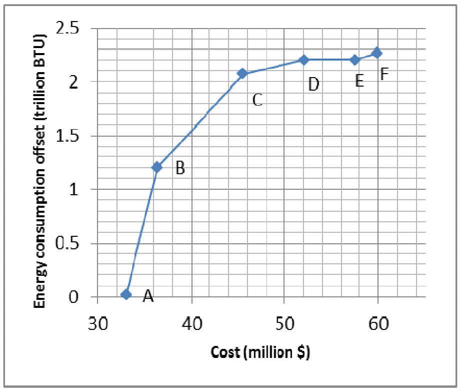

Figure 5. Cost and energy consumption tradeoff curve.

Solutions B, C and D are noninferior solutions in which none of the three objectives reaches its minimum possible value. These "intermediate" solutions try to accommodate all three objectives based on the provided objective weights. The combinations of MSWM options for these intermediate solutions are also different from the least cost, minimum energy and minimum CO_2 solutions. Fig. 6 shows the selected MSWM options for solution B. This solution is trying to depart from the least cost solution A and provides a greater weight to both the energy and emissions objectives. Therefore, the selection of technologies includes, in addition to the mandatory recycling, Refuse Derived Fuel (RDF) to try to offset energy consumption via the generation of energy from waste, and to avoid CO_2 emissions by reducing the amount of waste to be disposed at landfills. Fig. 7 shows the selected MSWM options for solution C, which increments the weight values again for both the energy and emissions objectives. The selection of technologies includes now both

RDF and waste to energy to try to improve the environmental objectives at the expense of cost. Finally, and following the same reasoning, Fig. 8 shows the selected options for solution D, which now includes only waste to energy and RDF, and excludes yardwaste composting.

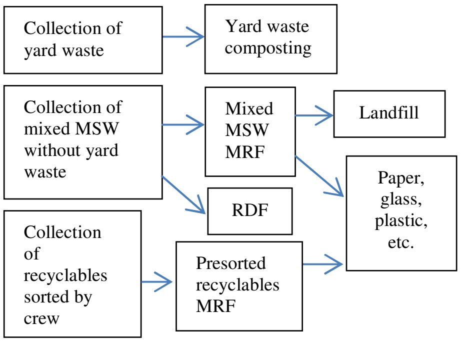

Figure 6. Management options for Solution B

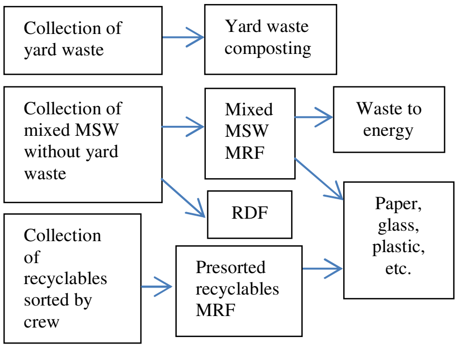

Figure 7. Management options for Solution C

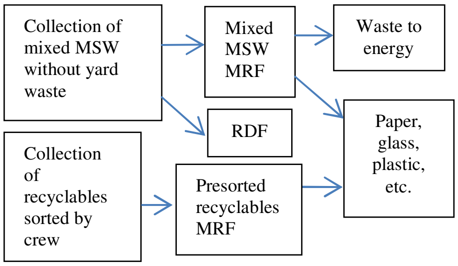

Figure 8. Management options for Solution D

E. Multi-objective Fuzzy Optimization

The multi-objective fuzzy optimization method simplified the task of choosing a unique solution among possible solutions from the noninferior solution set determined previously. Because the different objectives use very different measuring units, the weights used in the traditional weighted sum method were counter-intuitive. By using the membership functions in the multi-objective fuzzy optimization method, the objective values are normalized to comparable dimensionless units.

Table 3 shows the results obtained after running a number of scenarios by varying the relative importance given to the objectives. The relative importance given to the objectives through the weights had a direct effect on their reported degree of achievement. The greater the relative importance given to an objective, the greater the degree of achievement was for that objective in the reported solution.

Scenario 1 was obtained by assigning five times more importance to cost than to the other objectives. Cost obtained a degree of achievement of 87% while the other two objectives obtained degrees of achievement below 60%. Scenario 2 assigned five times more importance to energy than to cost, and the degree of achievement obtained for energy was 97% in detriment of cost. Since energy and CO_2 are non-competing objectives, the degree of achievement for CO_2 was also high. Scenario 3 assigned five times more importance to CO_2 than to cost, and the degree of achievement obtained for CO_2 was 97% in detriment of cost. Again, since energy and CO_2 are non-competing objectives, the degree of achievement for energy was high.

The combinations of MSWM options corresponding to the solutions obtained in these 3 scenarios are also different from the least cost, minimum energy and minimum CO_2 solutions, and from the solutions B, C and D obtained in the previous section. The fuzzy multi-objective linear programming method presented here allows for the exploration of the decision space, as different combinations of management options are likely to be selected when new noninferior solutions are chosen.

TABLE III. SUMMARY OF SOLUTIONS FROM FUZZY MULTI-OBJECTIVE OPTIMIZATION

Scenario	Relative importance	Objective values (1=cost, 2=energy, 3=CO2)	Degree of achievement $(1-\lambda_i)$
1	w1 =5	Z1=36.4 M$	λ1=87%
	w2=1	Z2=1.21 trillion BTU	λ2=53%
	w3=1	Z3=137.0 million lbs	λ3=60%
2	w1 =1	Z1=51.1 M$	λ1=33%
	w2=5	Z2=2.20 trillion BTU	λ2=97%
	w3=1	Z3=194.8 million lbs	λ3=95%
3	w1 =1	Z1= 53.3 M$	λ1=25%
	w2=1	Z2= 2.20 trillion BTU	λ2=97%
	w3=5	Z3= 197.6 million lbs	λ3=97%

IV. DISCUSSION

Municipal Solid Waste Management (MSWM) is a difficult task when a number of optional technologies and management processes are available to choose from. MSWM simulation is crucial to help generate and analyze multiple and different waste management scenarios. It allows the analyst to choose from a number of optional objectives. Initially, optimal solutions were found for a waste management scenario under three different objectives: minimum cost, minimum carbon dioxide emissions and minimum energy consumption. The combination of unit processes associated with these optimal solutions was very different from one another. While the minimum cost solution included collection of mixed waste and processing of recyclables in a mixed waste MRF, the other two scenarios included additional recyclable collection and processing options. Only the minimum cost solution included yardwaste composting. Both minimum energy and minimum carbon dioxide emissions included waste-to-energy as a means of offsetting energy consumption with the electricity generated at the waste-to-energy plant. The mandated diversion rate of 15% was satisfied by all three scenarios in different ways. The minimum cost solution diverted waste by composting yardwaste and recycling. The other two scenarios relied on recycling only. Both minimum energy and minimum carbon dioxide used waste-to-energy to offset energy consumption and emissions from regular unit processes.

Decision makers may also want to find MSWM options while satisfying multiple objectives at once. The MSWM simulation allowed us to perform multiple-objective optimization to find interesting and different solutions from those obtained by minimizing on one objective at a time. The weighted sum methodology is a simple and reliable way to obtain non-inferior solutions from the Pareto optimal set. Each individual non-inferior solution represents a compromise between the competing objectives, and will favor one objective with respect to the other depending on the relative weights used. The convexity of the Pareto frontier means that, when moving from one non-inferior solution to another, an improvement in one of the objectives will represent degradation in the other objective. Non-inferior solutions represent interesting multi-objective scenarios for the decision maker to consider. They represent different points in the objective space, combining multiple objectives by means of the weighted sum methodology. They also represent potential different points in the decision space, which can provide valuable information about a variety of MSWM strategies. A drawback of the traditional weighted sum methodology is the potential disparity in the values of used weights, due to the different measurement units used by the objectives. The fuzzy multi-objective methodology presented here provides the modeler with the possibility to select a noninferior solution and quantify the degree of achievement for each of the competing objectives. By normalizing the objective values via the membership functions, it eliminates the inconvenience of having dissimilar measurement units. It offers a method to manage imprecision and uncertainty in the objectives via fuzzy membership functions defined based on the worst achievement level and the best achievement level for each objective. By changing the relative level of importance on the objectives, the user can explore the noninferior set and have a quantifiable means to rank the degree of optimization obtained for each individual objective. The user may also experience different combinations of waste management options, as changes in the solution space may imply changes in the decision space.

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